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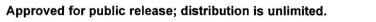
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**UNSTEADY FLUID MOTION IN LIQUID FILLED PROJECTILES** 

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For spinning and coning cylinders with liquid payloads, we established the Unified Approach based on our spatial					
eigenfunction expansion method as a powerful tool for studies of two types of instability: instability caused by large viscous					
stresses at low Reynolds' number	ers and instability owing to reson	ance with inertial wave	s at mediu	m and high Reynolds	
numbers. The applications of the Unified Approach to completely-filled cylinders, partially-filled cylinders, cylinders with a					
central rod, and cylinders with two immiscible fluids successfully demonstrated interesting physical mechanisms such as					
resonances and produced useful results for future experiments and designs. We also performed inviscid analyses and					
computations by spectral collocation method as complementary studies for the Unified Approach. We developed unsteady					
three-dimensional Navier Stokes codes and employed them to simulate spin-down experiments. We made progress in the					
moment calculation for silicone	fluids, and investigated multiple	states and the stability	of flows i	n loosely coiled pipes.	
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#### **PREFACE**

The work described in this report was performed under Contract No. DAAA15-89-K-0002. This work was started in December 1990 and completed in September 1993.

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# 1 Objectives

Following are the original research goals for this three-year contract:

- (1) Analyze the resonance phenomena between cylinder motion and inertial waves at high and medium Reynolds numbers and in eccentric and elliptic cylinders.
- (2) Study the unsteady motion caused by time-dependent boundary conditions, in particular the axisymmetric spin-up during firing and the ramp-up of the nutation angle at launch.
- (3) Analyze unsteady (cellular or turbulent) motions that develop at high Reynolds numbers under the quasi-steady conditions during flight.
- (4) Develop methods for the analysis and study the effect of partial-filled (liquid-air) and completely-filled with two immiscible fluids.

The emphasis on the various topics shifted during the working period to better support ongoing experiments and flight tests at CRDEC. We achieved most of our original objectives and investigated some additional topics. The details of these achievements are described in the following section.

### 2 Achievements

# 2.1 Unified Approach for Stability Analysis

Spin-stabilized projectiles can experience two types of dynamical instabilities resulting from the motion of a liquid payload. The first type is caused by resonance with inertial waves of high Reynolds number at certain aspect ratios and coning frequencies. The common methods for the analyses of this type of instability is the ISW method (improved Stewartson-Wedemeyer method) developed at BRL (Ballistic Research Laboratory), which applies the traditional boundary layer theory. The second type of instability is generated by viscous stresses on the walls of cylinders and is pronounced at low to medium Reynolds numbers for a wide range of aspect ratios and coning frequencies. During the last decade, this type of instability was investigated either by various numerical methods which solve the steady 3D Navier-Stokes equations

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for low Reynolds numbers at relatively high computational costs or by the spatial eigenfunction method (HSG method) developed at BRL (P. Hall, R. Sedney and N. Gerber 1987). Some of the numerical methods can be applied for Reynolds numbers up to about 1000 while the HSG method works up to about 2500. However, these methods are not well suited to cover the range of medium Reynolds numbers. Moreover, none of them can handle two-fluid cases in which a small amount of low-viscosity fluid (with Re of order 10<sup>6</sup>) is an additive for the high-viscosity fluid payload (with Re of order 10 to 100). Obviously, it is desirable to have a single method which can deal with all Newtonian flows in different cylinder configurations, thus overcoming the limitations of other methods and simplifying the analyses of the two types of instability. This goal has now been achieved by the "Unified Approach", which is based on our spatial eigenfunction expansion method (R. Li and Th. Herbert, 1988; M. Selmi, R. Li and Th. Herbert, 1992)

### 2.1.1 Spatial Eigenfunction Expansion Method

The spatial eigenfunction expansion method is based on observations from previous numerical computations: in the control volume analysis, the moments depend essentially on the axial velocity component. When expressing the axial velocity in a Fourier series in the azimuthal direction, only the fundamental component is needed for accurate calculations of both yaw and roll moments and for good estimates of the pitch moment. By manipulating the linearized Navier-Stokes equations, we derive a single sixth-order partial differential equation for the axial velocity. Since the governing equation and the three boundary conditions at each end wall are homogeneous, the axial structure of the solution can be expanded in spatial eigenfunctions (complex trigonometric functions). By applying the Fourier analysis in the azimuthal direction and separation of variables, the three-dimensional problem is reduced to one dimension, and the radial structure of the solution can be expressed in Bessel functions. The sixth-order partial differential equation is reduced to an eigenvalue problem. Because the solutions are expressed as sums of the products of well known functions, only the eigenvalues need to be determined by numerically solving a closed form characteristic system of equations. The expansion coefficient are found by satisfying the boundary conditions at the side walls using a collocation method.

Our spatial eigenfunction expansion method (Unified Approach) is effi-

cient in computing moments and flexible in treating partial-filled cases and two-fluid cases. Through years of improvement, the method makes it possible to compute the liquid-induced moments from low Reynolds numbers (almost zero) to high Reynolds numbers (up to order of  $10^6$ , for example, Re=2263000) for completely-filled cylinders, partially-filled cylinders, cylinders with a central rod, and cylinders with two immiscible fluids (R. Li and Th. Herbert, 1992). By comparison, the HSG method employs the spatial eigenfunctions in the radial direction, and these functions must be precalculated by a numerical method, which restricts its applications to  $Re \leq 2500$ .

# 2.1.2 Results for Completely-Filled Cylinders, Partially-Filled Cylinders and Cylinders with Central Rod

For completely-filled cylinders, the results of computations by the Unified Approach agree well with all known results obtained previously by different methods at low and large Reynolds numbers. According to the request of CRDEC, we ran all the examples selected to be included in the Liquid-fill Handbook. Our results agree well with results obtained by the ISW nethod at high Reynolds numbers (Re=226300 and Re=180700) and by the HSG method at low Reynolds numbers (Re=226.3, Re=180.7, Re=22.63, Re=18.07) (D. J. Weber et al. 1992). The Unified Approach can easily cover the range of medium Reynolds numbers. Our results for completely-filled cylinders allow to properly choose the aspect ratio to avoid the resonance for medium and high Reynolds number flows.

In practice, the aspect ratio may not be allowed to change. In such cases, there are two simple approaches to intentionally alter the liquid-induced moments to avoid the resonance. The first approach is to fill the cylinder partially with the fluid during the production. This approach was experimentally studied by Miller (1981). The second approach is to insert a central rod inside the cylinder. This approach was suggested by Murphy et al. (1989) and some analyses were done at BRL. Both approaches introduce the fill ratio as a new parameter. Our Unified Approach enables us to see the remarkable maxima acquired by the moments at critical aspect ratios, coning frequencies, and fill ratios for a wide range of Reynolds numbers. We have studied a partially-filled cylinder (Re=12624) to compare the Unified Approach with the ISW method. Our Unified Approach produces almost the same results as the ISW method (R. Li and Th. Herbert, 1992). Through intensive studies,

we obtain some important conclusions for practical designs. One of these conclusions is that for partially-filled cylinders, the fill ratio is crucial to the onset of resonance, especially in the case of low viscosity fluid. Reducing the amount of fluid inside the cylinder may not always reduce the roll moment if the fill ratio is not chosen correctly. Another conclusion is that the central rod configuration is more effective in eliminating resonance than a partially-filled cylinder with the same fluid volume.

# 2.1.3 Results of Cylinders with Two Fluids

The superiority of the Unified Approach over other current methods is the capability to efficiently study two-fluid cases. The studies of two immiscible fluids of different density and viscosity in a completely-filled cylinder are motivated by the efforts to reduce the viscous stress effect at low Reynolds numbers. It was suggested that a small amount of low-viscosity additive in contact with the wall may lubricate the core fluid of high viscosity and thus reduce the roll moment. Because of the existence of two fluids, we have to add some new parameters for determining the instabilities: the Reynolds number of the outer fluid (additive) Re1, the Reynolds number of the inner (core) fluid  $Re_0$ , the density ratio  $\rho_0/\rho_1$ , and the fill ratio,  $V_1/V$ , the ratio of the volume of outer fluid to the volume of the whole cylinder. Our comprehensive studies by using the Unified Approach have generated many interesting results which cover a wide range of aspect ratios, fill ratios, density ratios, and combinations of  $Re_0$  and  $Re_1$ . For the optimum reduction of the roll moment, the density ratio must be less than but close to 1, and the additive must be much less viscous than the core fluid (M. Selmi, 1991). Some of our recent results show that using low-viscosity additive as a lubricant for the high-viscosity fluid to reduce the liquid-induced moment may or may not work depending on the aspect ratio (R. Li and Th. Herbert, 1992). The lubrication mechanism of the low-viscosity additive can only work for aspect ratios far from resonance. In these cases, a small amount of additive can greatly reduce the roll moment, sometimes to about one-half or even less. However, for aspect ratios close to resonance, a small amount of additive will increase the roll moment.

Since the Unified Approach can be applied to very high Reynolds numbers, we are able to perform simulations for actual experiments with water as an additive. Our recent computations show that for a Newtonian fluid, the

Unified Approach produces results in qualitative and quantitative agreement with the experimental findings. This predictive capability permits reliable flight simulations.

# 2.1.4 Complementary Studies: Inviscid Solutions and Solutions by Spectral Collocation Method

As a double-check for the studies of the resonance phenomena of high Reynolds number flows, the inviscid equations for the cylinders with completely-filled, partially-filled, central rod, and two fluids are also solved. Compared with the viscous case, the closed-form solutions of these inviscid equations are much easier to obtain. All the inviscid results are consistent with the results of high Reynolds number flows, thus providing the limits of the aspect ratio, fill ratio, and density ratio that cause resonance. These results strengthen our understanding of the inviscid nature of the resonance phenomena in high Reynolds number flows.

Although it is successful in calculating the moments for various configurations, the Unified Approach (spatial eigenfunction expansion method) is based on the linearized Navier-Stokes equations and linearization neglects the mean flow distortion which has a minor contribution to the pitch moment. To investigate the effect of the discarded nonlinear terms on the pitch moment, the spectral collocation method has been used to solve the three-dimensional full Navier-Stokes equations. The spectral collocation method is based on representing the flow quantities by Chebyshev series in the axial and radial directions and a Fourier series in the azimuthal direction. This method was successfully applied to the case of completely-filled cylinders before (Herbert and Li 1990) and extended to partially-filled cylinders and cylinders with a central rod within this contract. The Reynolds number is up to 1000. The method not only provides us the moments but also the details of the threedimensional fluid motion which can be visualized on a graphic workstation. The comparisons of the roll and yaw moments with the eigenfunction expansion method indicate that nonlinear effects are negligible, especially when the flow parameters do not lead to resonance. Even in the cases of resonance the differences are insignificant for all practical applications. The differences for the pitch moment are increased as the Reynolds number increases. However, these differences are small, in general less than 4 percent.

### 2.2 Numerical Studies of Unsteady Flows

During the last decade, several Navier-Stokes solvers for the numerical study of viscous incompressible flow in a spinning and nutating cylinder were developed. All these solvers were designed for the calculations of steady states only. We developed Navier-Stokes solvers for the analysis of unsteady three-dimensional flows in circular cylinders. The codes use the time-accurate finite-difference method for the velocity-vorticity formulation of the equations. We also derived and implemented formulas to calculate the moments for the unsteady motion based on the volume integral approach.

As the first application of the solvers, we simulated the free spin-down case at fixed nutation angle for high viscosity flows. The time history of the spinning measured from the experiments is converted to the time-dependent boundary conditions. In the previous experimental work, the moments obtained from such "quasi-steady" runs were generally assumed to be close to the results for the steady states. Our simulations verify this assumption. The calculations show that for high viscosity flows, the unsteady flow field adapts quickly to the slowly changing boundary conditions such that the contributions of the time derivative of the velocity deviation part to the moments are negligible. When we compare the moments of steady with those of the spin-down at "instantaneous" Reynolds numbers, we conclude that the moments obtained from the "quasi-steady" spin-down experiments can be regarded as those for steady states if the roll moment is corrected by taking the spin deceleration of rigid-body motion into account.

This solver also can be used for the calculation of the steady states. It is efficient for low Reynolds numbers (Re < 200). The results are in good agreement with those obtained by the spectral collocation method.

# 2.3 Moment Calculation for Silicone Fluids

The high viscosity fluid inside a spinning and nutating cylinder can generate large despin moments. M. C. Miller and his workers at CRDEC found that at low Reynolds numbers, the fluids (Silicone 200) used in the experiments often produce despin moments substantially different from those for Newtonian fluids of the same viscosity. Experiments performed by R. P. Tytus (1989) confirm that Silicone 200 at high viscosity exhibits non-Newtonian behavior and indicate that the differential model may be appropriate for

these silicone fluids. Tytus applied Rosenblat's formula for the differential model (1986) with the measured fluid parameters to calculate the moments in a spinning and nutating cylinder. His results, however, are not in good agreement with experimental data. In some cases, the calculated moments are several times larger than the experimental data and have non-physical peaks as the Reynolds number changes. We reviewed the previous work by Rosenblat and Tytus and found that good agreement between the theoretical formula and the experimental data for the despin moment can be achieved if the parameters in the formula are chosen properly. Furthermore, we proposed a new differential model for silicone fluids. The formula derived from this proposed model produces physically more reasonable results for the moments which agree not only qualitatively but also quantitatively with the experimental data. The new model also generates improved results for the experimental values of the normal stress coefficients and viscosity of the simple shear flow. To further support and improve the new model requires more detailed experiments on the non-Newtonian behavior of the silicone fluids and direct measurements of the pitch moment.

### 2.4 Computer Visualization of Flows

Visualizations of the flows were performed on graphics workstations. The visualizations highlighted the differences between flows at low and high Reynolds numbers and the contrasts between viscously damped and near-inviscid resonant flows. The data show the fluid motion in completely-filled and partially-filled cylinders and the computational analogue to the void observations conducted earlier at CRDEC.

# 2.5 Multiple States and Stability of Flows in Loosely Coiled Pipes

The goal of this study is the development of tools for the stability analysis of 3D flows which depend on two variables. The incompressible Newtonian fluid in a coil pipe is of fundamental importance for the design of heat exchangers, e.g. in chemical processing. The governing equations for large coil radii are solved by a spectral collocation method. Our results confirm the occurrence of three solution branches and two turning points at high Dean numbers. Within the class of steady symmetric and anti-symmetric disturbances, the

first branch is stable up to high Dean numbers, while the second and the third branches are always unstable. The friction factors are found in agreement with experimental correlations but different from previous perturbation solutions. We have analyzed the nonphysical singularity for imaginary Dean numbers which is relevant to the convergence of perturbation series.

### 3 Personnel

The following personnel were partly supported under this contract:

Thorwald Herbert, Professor, Principal Investigator

Rihua Li, Research Associate

Selmi Mohamed, Graduate student (Ph.D. level)

Hsi-Yung Feng, Graduate student (M.S. level)

Charlotte R. Herbert, Systems Programmer 2

### 4 Publications

The following publications, reports, and communications acknowledge the support by CRDEC.

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M. Selmi and Th. Herbert, "Resonance Phenomena in Viscous Fluids inside Partially Filled Spinning and Nutating Cylinders," Submitted to *Phys. Fluids A*, (1992).

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Th. Herbert and H. Y. Feng, "Flows in a Loosely Coiled Pipe and Their Stability, Bull. Amer. Phys. Soc., Vol. 34, pp. 2319 (1989).

# 5 Conference Papers and Presentations

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D. J. Weber, M. J. Nusca, R. Li and Th. Herbert, "Overview of the Illustrated Examples Section of the Liquid-Fill Projectile Design Handbook," Proc. 1992 U.S. Army Edgewood Research, Development and Engineering Center Scientific Conference on Chemical Defense Research, Nov. 17-20, 1992, Aberdeen Proving Ground, MD.

R. Li and Th. Herbert, "Progress in the Moment Calculation of a Spinning and Nutating Cylinder with Silicone Fluid," Proc. 1991 U.S. Army Chemical Research, Development and Engineering Center Scientific Conference on Chemical Defense Research, Nov. 19-22, 1991, Aberdeen Proving Ground, MD.

M. Selmi, "Effect of Low Viscosity Additives on the Moments Caused by Viscous Fluids in a Spinning and Coning Cylinder," Proc. 1991 U.S. Army Chemical Research, Development and Engineering Center Scientific Conference on Chemical Defense Research, Nov. 19-22, 1991, Aberdeen Proving Ground, MD.

Th. Herbert, R. Li and M. Selmi, "Analysis and Visualization of the Flows in a Spinning and Nutating Cylinder," Proc. Workshop on Problems of Rotating Fluids, Army High Performance Computing Research Center (AHPCRC), Minneapolis, MN, April 22-23, 1991.

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H. Feng and Th. Herbert, "Multiple States and Stability of Flows in Loosely Coiled Pipes," Proc. 1990 U.S. Army Chemical Research, Development, and Engineering Center, Scientific Conference on Chemical Defense Research, Nov. 13-16, 1990, Aberdeen Proving Ground, MD.

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Hsi-Yung Feng, "Multiple States and Stability of Flows in Loosely Coiled Pipes," M.S. Thesis, Department of Mechanical Engineering, The Ohio State University, 1990.

Mohamed Selmi, "Resonance Phenomena in Viscous Fluid Configurations inside a Spinning and Coning Cylinder," Ph.D. Thesis, Department of Mechanical Engineering, The Ohio State University, 1991.

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